The Optimal Spatial Distribution of Small and Large Scale Fisheries in Chile

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Saint Andrews
February 26 2009

The problem

A marine space along the Chilean coast split :

1. coastal space :   high reproduction size

The problem

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2. open sea : lower reproduction size

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1. small-scale fishermen : artisanal fishery,
   - near the coast : 5 nautical miles

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   - biomass transfer

Two users :

1. small-scale fishermen : artisanal fishery,
   - near the coast : 5 nautical miles
2. large-scale fishermen : industrial fishery,
   - only at some distance from the coast

About two modelizations

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1. Global
   - z : global biomass. x_1, x_2 : biomass zones 1,2.
   - $\dot{z} = f(z, p) \quad z = x_1 + x_2$

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2. Patches
   - $\dot{x}_i = f(x_1, x_2, p)$

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$$\dot{z} \quad = \quad \gamma z(1 - \frac{z}{K})$$

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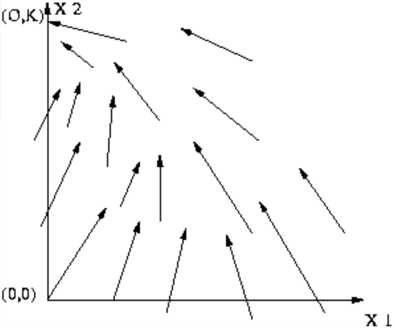
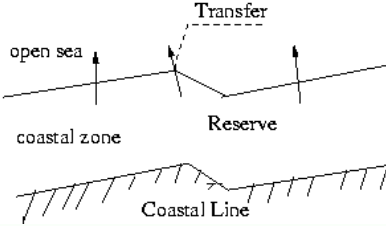
- $(0,0)$ saddle point, $\ (0,K)$ is stable

The model without harvest





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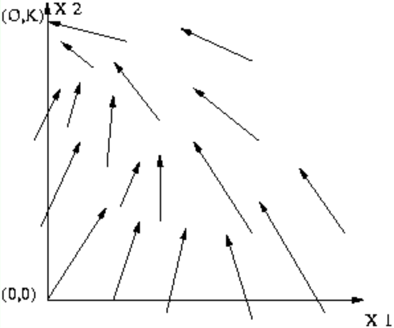
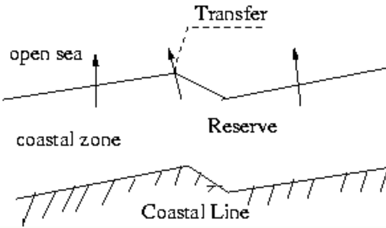
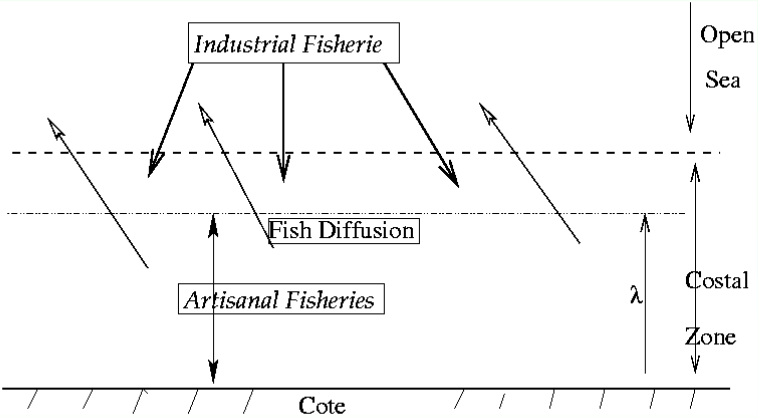




The coastal zone acts as an reserve.

The model with harvest



The model with harvest

$$\begin{aligned}
\dot{x}_1 &= \lambda\gamma(x_1+x_2)\left(1-\frac{1}{K}(x_1+x_2)\right) - bx_1 - q_1E_1x_1 \\
\dot{x}_2 &= (1-\lambda)\gamma(x_1+x_2)\left(1-\frac{1}{K}(x_1+x_2)\right) + bx_1 - q_2E_2x_2
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$\lambda \in [0,1]$

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The revenue :

- $p_i, c_i, q_i \;\; i = 1, 2$
- $(p_2 q_2 x_2(t) - c_2)\, E_2(t) + (p_1 q_1 x_1(t) - c_1)\, E_1$

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$$\int_0^{+\infty} e^{-\delta t}\left[(p_2q_2x_2(t) - c_2)\,E_2(t) + (p_1q_1x_1(t) - c_1)\,E_1\right]dt$$

The Optimal Control problem

$$\max_{E_2(.),\lambda} \quad \int_0^{+\infty} e^{-\delta t} \left[\left(p_2 q_2 x_2(t) - c_2 \right) E_2(t) + \left(p_1 q_1 x_1(t) - c_1 \right) E_1 \right] dt$$

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$0 \leq E_2(t) \leq E_M$

$\lambda \in [0, 1]$

Equivalent problem

$$\max_{(x_1(.),x_2(.))\in C} \int_0^{+\infty} e^{-\delta t} l(x_1(t), x_2(t), \dot{x}_1(t), \dot{x}_2(t))dt$$

where

$$\begin{aligned} l(x,\dot{x}) = \; & (p_2 - \frac{c_2}{q_2 x_2})[\gamma(x_1 + x_2)(1 - \frac{x_1+x_2}{K}) - q_1 E_1 x_1 - \dot{x}_1 - \dot{x}_2] \\ & + (p_1 q_1 x_1 - c_1)E_1 \end{aligned}$$

Equivalent problem

$$\max_{(x_1(.),x_2(.))\in C} \quad \int_0^{+\infty} e^{-\delta t} I(x_1(t), x_2(t), \dot{x}_1(t), \dot{x}_2(t))dt$$

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$$C = \qquad \{x(.) \in BC^1([0,\infty]), \textit{with constraints}\}$$

Equivalent problem

Euler-Lagrange Equation :

$$l_x(x,\dot{x}) - \frac{d}{dt} l_{\dot{x}}(x,\dot{x}) + \delta l_{\dot{x}}(x,\dot{x}) = 0$$

Proposition

$$\begin{aligned}
\dot{x}_1 &= x_2\left(\tfrac{p_2 q_2}{c_2} x_2 - 1\right)\left[\gamma\left(1 - 2\tfrac{x_1+x_2}{K}\right) - \delta\right] + \gamma(x_1+x_2)\left(1 - \tfrac{x_1+x_2}{K}\right) \\
&\quad -q_1 E_1 x_1 \\
\dot{x}_2 &= -x_2\left(\tfrac{p_2 q_2}{c_2} x_2 - 1\right)\left[\gamma\left(1 - 2\tfrac{x_1+x_2}{K}\right) - \delta\right] \\
&\quad +q_1 E_1 x_2\left[\tfrac{q_2}{c_2}(p_2 - p_1) x_2 - 1\right]
\end{aligned}$$

Stationary solutions

Proposition case $p_1 = p_2$
$\gamma > q_1 E_1$ and if there exists $\alpha > 0$ s.t.

$$\begin{array}{rcl} q_1 E_1 & < & \frac{\gamma+\delta}{(2+\alpha)} \\ \frac{p q_2 K}{c_2} & > & (1 + \frac{1}{\alpha}) \left(\frac{\gamma}{\gamma - q_1 E_1} \right) \end{array}$$

then there exists exactly one stationary point $x^* = (x_1^*, x_2^*)$ s. t. $x_1^* > 0$ and $x_2^* > 0$,

$$\begin{array}{rcl} x_2^* & = & \frac{c_2(\gamma+\delta-q_1 E_1)}{p q_2(\gamma+\delta-2q_1 E_1)} \\ x_1^* & = & \frac{K}{\gamma}(\gamma - q_1 E_1) - \frac{c_2(\gamma+\delta-q_1 E_1)}{p q_2(\gamma+\delta-2q_1 E_1)} \end{array}$$

Associated controls

Proposition

Associated to the steady states x_1^*, x_2^*, the values of the two controls are :

$$\begin{aligned} E_2^* &= \frac{q_1 E_1}{q_2} \\ \lambda^* &= \frac{q_1 E_1 + b}{q_1 E_1 \left(1 + \frac{x_2^*}{x_1^*}\right)} \end{aligned}$$

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- $0 \leq E_2^* \leq E_M$
- If $b = 0$ then $0 \leq \lambda^* < 1$

Proposition An interior solution for the optimal artisanal reserve size exists if the following condition holds true

$$b < q_1 E_1 \frac{x_2^*}{x_1^*}$$

Stability property

Proposition

If $\gamma > q_1 E_1$ and $\gamma > q_2 E_2$, then the equilibrium x_1^*, x_2^* is stable.

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Conclusion

With low transfer, it is optimal to allow the industrial fleet to develop activities in the growth area.

Numerical application

Biological quantities :		**Harvesting parameters :**	
Instantaneous growth	$\gamma = 0.16$	Artisanal capturability	$q_1 = 10$
Carrying capacity	$K = 100$	Industrial capturability	$q_2 = 0.1$
Biomass Transfer	$b = 0.03$	Artisanal effort	$E_1 = 0.01$

Economical parameters :	
Price per unit harvest	$p = 100$
Cost per unit of artisanal effort	$c_1 = 10$
Cost per unit of industrial effort	$c_2 = 20$
Discount factor	$\delta = 0.06$

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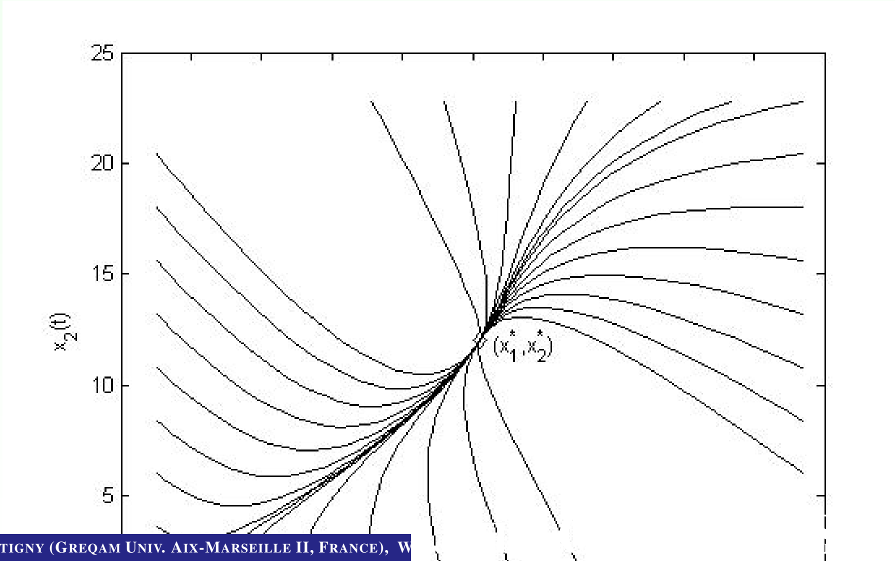
Discount factor	$\delta = 0.06$

$$x_1^* = 25.5 \qquad E_2^* = 1$$
$$x_2^* = 12 \qquad \lambda^* = 0.884$$
$$I(x_1^*, x_2^*, 0, 0) = 355$$

Trajectories



The case $\lambda = 1$

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Euler-Lagrange equations :

$$\begin{aligned} & \frac{c_1}{q_1 x_1^2} \left(\gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - bx_1 \right) \\ & + (p - \frac{c_1}{q_1 x_1})(\gamma - b - \delta - 2\gamma \frac{x_1 + x_2}{K}) + (p - \frac{c_2}{q_2 x_2})b = \ 0 \end{aligned}$$

$$\frac{c_2 b x_1}{q_2 x_2^2} + (p - \frac{c_1}{q_1 x_1})(\gamma - 2\gamma \frac{x_1 + x_2}{K}) - (p - \frac{c_2}{q_2 x_2})\delta = \ 0$$